



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2015

MT 2501/MT 2500 – ALGEBRA, ANALY.GEO, CALCULUS – PAPER-II

Date : 27/04/2015
Time : 09:00-12:00

Dept. No.

Max. : 100 Marks

PART-A

Answer ALL questions:

(10 x 2 =20)

1. Evaluate $\int \sin^4 x dx$.
2. Evaluate $\int (\log x)^2 dx$.
3. Solve $\frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{\frac{1}{2}} = 0$
4. Solve $(D^2 - 5D + 6)y = 0$.
5. Test the convergence of series $\sum_{n=0}^{\infty} \frac{n^3 + 1}{2^n + 1}$.
6. State the Cauchy's root test.
7. Write the expansion of $(1-x)^{-\frac{1}{2}}$.
8. Find the coefficient of x^n in $1 + \frac{b+ax}{1!} + \frac{(b+ax)^2}{2!} + \dots + \frac{(b+ax)^n}{n!} + \dots$
9. Find the angle between the planes $x + y + 2z - 3 = 0$ and $2x - y + z - 6 = 0$.
10. Find the equation of sphere with centre $(-1, 2, -3)$ and radius 3 units.

PART-B

Answer any FIVE questions:

(5 x 8=40)

11. Find the area of the surface of the solid generated by rotating the cardioid $r = a(1 + \cos \theta)$ about its the line of symmetry.
12. Evaluate $\int x^4 (\log x)^3 dx$.
13. Solve $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 x}{y^2}$
14. Solve $x dx + y dy - \left(\frac{x dy - y dx}{x^2 + y^2}\right) = 0$.
15. Examine the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$
16. Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \dots \infty$.
17. Show that $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \dots$
18. Show that $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find the equation of the plane containing them.

PART-C

Answer any TWO questions:

(2 x 20=40)

19. (a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2} + (\cos x)^{3/2}} dx$.

(b) Find the length of one loop of the curve $3ay^2 = x(x-a)^2$. (10+10)

20. (a) Solve $(D^2 + D + 1)y = \sin 2x$.

(b) Solve $xy'' - (2x+1)y' + (x+1)y = x^2e^x$. (10+10)

21. (a) When x is small prove that $\frac{(1-3x)^{-2/3} + (1-4x)^{-3/4}}{(1-3x)^{-1/3} + (1-4x)^{-1/4}} = 1 + \frac{3}{2}x + 4x^2$ approximately.

(b) Sum the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots$ (10+10)

22. (a) Prove that the series $1 + \frac{1}{2} \cdot \frac{a}{b} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{a}{b} \cdot \frac{(a+1)}{(b+1)} + \frac{1.3.5}{2.4.6} \cdot \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \dots$ is

convergent if $a > 0, b > 0$ & $b > a + \frac{1}{2}$.

(b) Find the shortest distance between $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. (10+10)

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